ME 243

Mechanics of Solids

Lecture 7: Beam deflections

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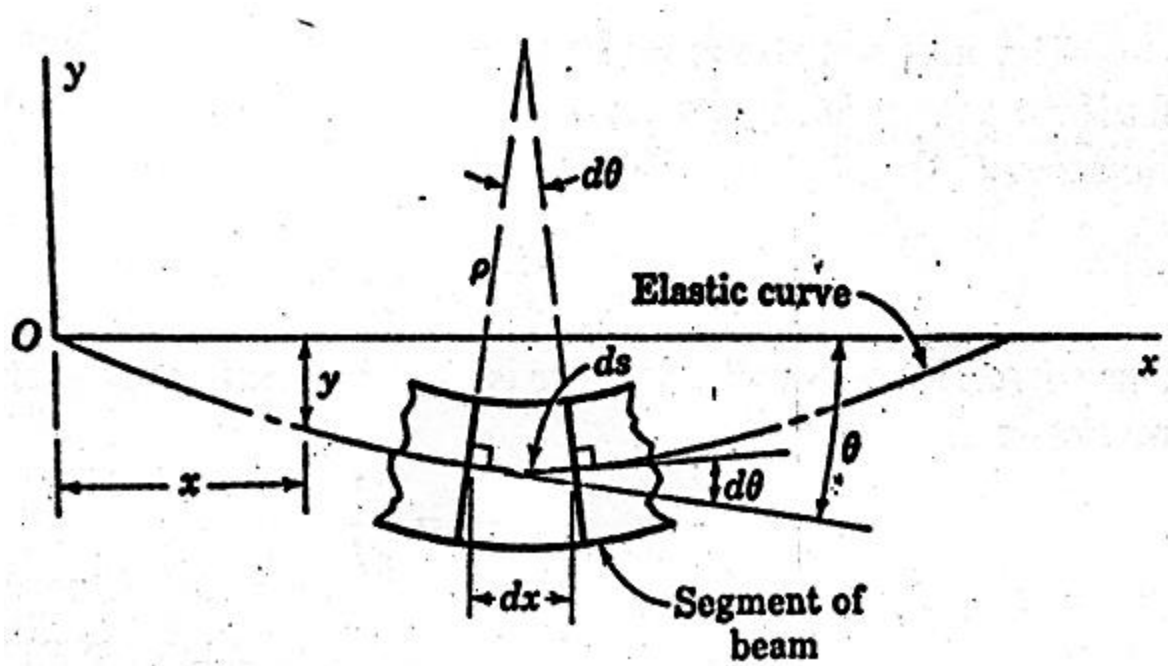
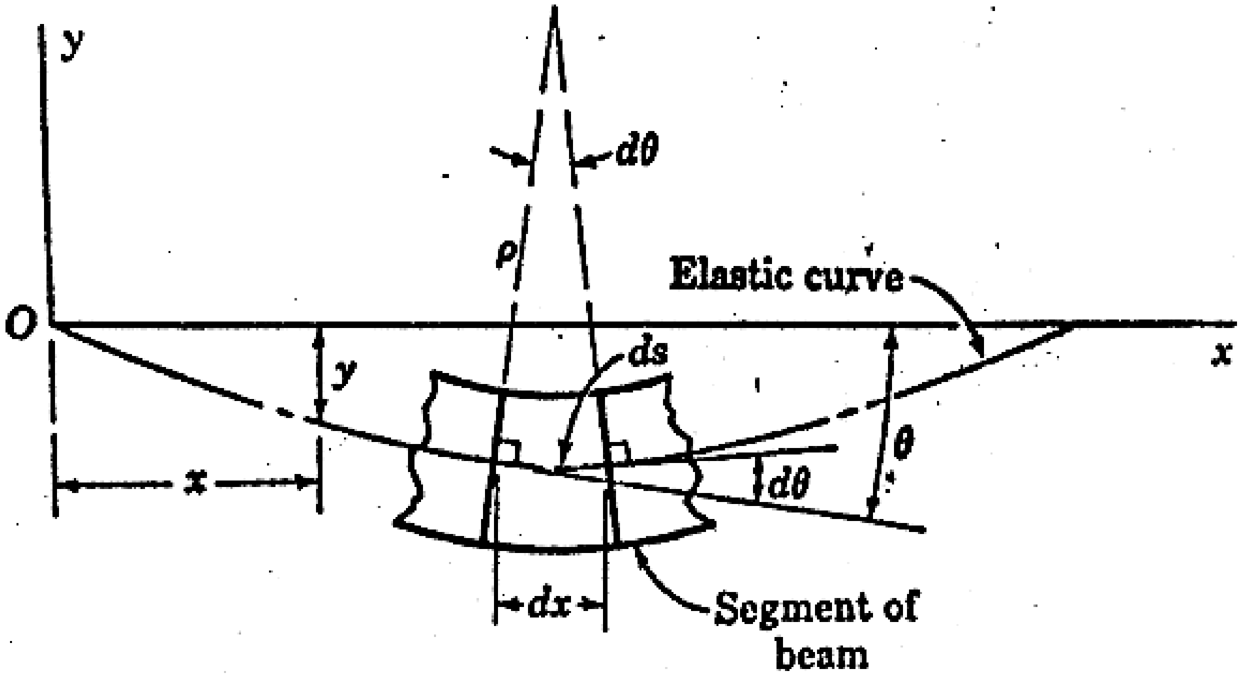
Double integration method

- The edge view of the neutral surface of a deflected beam is called the elastic curve of the beam.
- In this method, the equation of the elastic curve is first determined and then the deflection of beam at any point is determined.



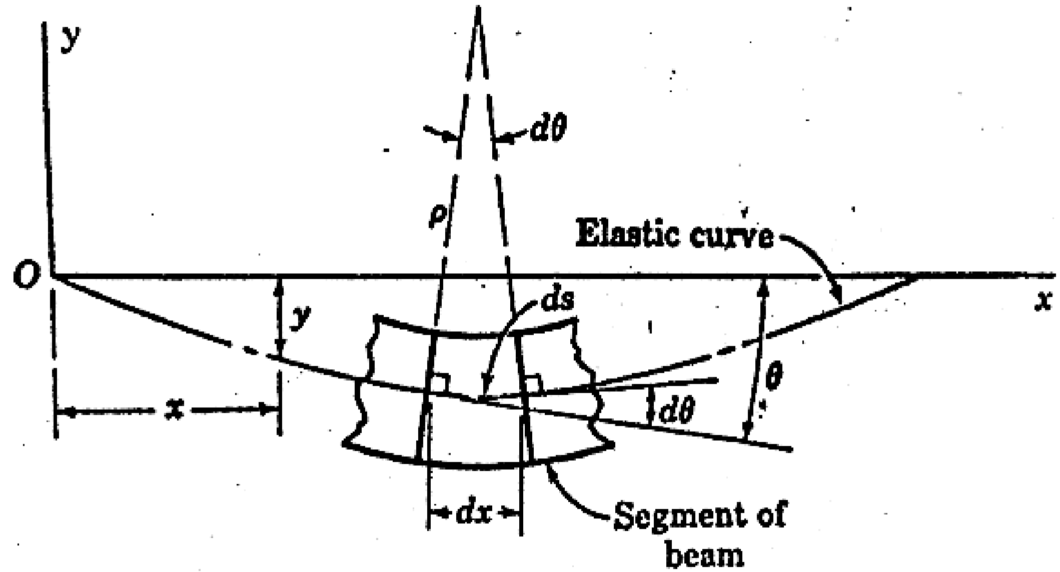
Double integration method



- The deflections are assumed to be so small that there is no appreciable difference between the original length of the beam and the projection of its deflected length. So the elastic curve is very flat and its slope at any point is very small.

The value of the slope,

$$\tan \theta = dy/dx.$$



As θ is so small,

$$\theta = \frac{dy}{dx}$$

$$\frac{d\theta}{dx} = \frac{d^2y}{dx^2}$$

Again , we can write, $ds = \rho \, d\theta$

Here, 'ρ' is the radius of curvature over the length 'ds'. As the elastic curve is flat, 'ds' is equivalent to 'dx'. So we can write

$$\frac{1}{\rho} = \frac{d\theta}{ds} \approx \frac{d\theta}{dx} \quad \text{or} \quad \frac{1}{\rho} = \frac{d^2y}{dx^2}$$

But we know, $$\frac{1}{\rho} = \frac{M}{EI}$$

So, we get, $$EI\frac{d^2y}{dx^2} = M$$

It is known as differential equation of the elastic curve of a beam.

The product ‘EI’ is called the flexural rigidity of the beam. It remains constant along the length of the beam.

Double integration method

We get, $$EI\frac{d^2y}{dx^2} = M$$

Integrating, we get,

$$EI\frac{dy}{dx} = \int M\,dx + C_1$$

It is the equation of the slope of the elastic curve.

Again, integrating, we get,

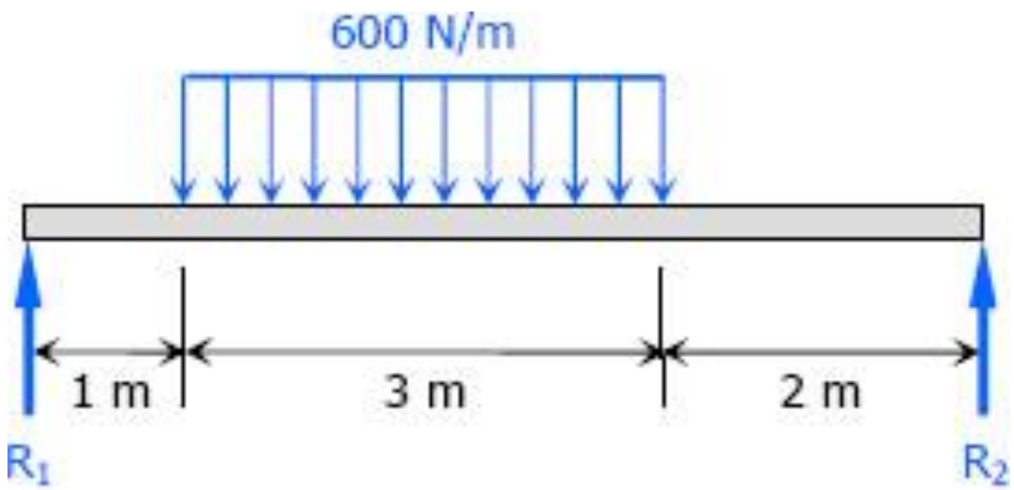
$$EIy = \iint M\,dx\,dx + C_1x + C_2$$

It is the deflection equation of the elastic curve of the beam.

- Moment is to be expressed as a function of x.
- Constants C_1 and C_2 can be obtained by using boundary conditions of the beam

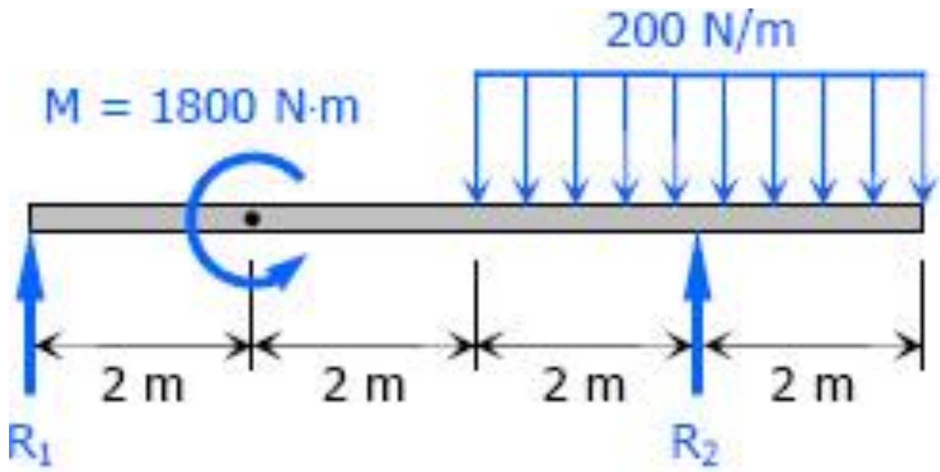
Problem# 612 (singer)

- Compute the midspan value of EIδ for the beam loaded as shown in Fig. by using double integration method.



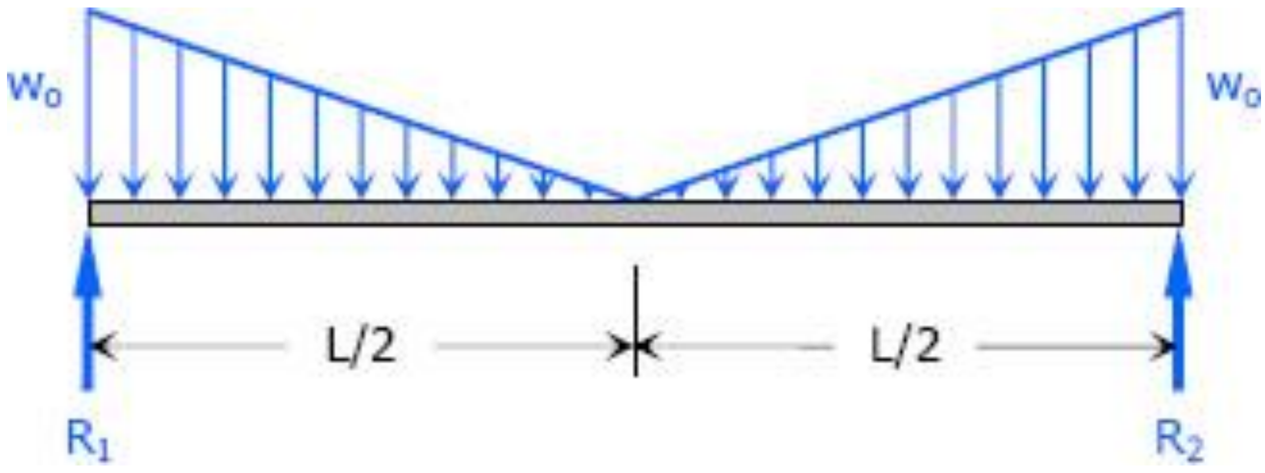
Problem# 619 (singer)

- Determine the value of EIy midway between the supports for the beam loaded as shown in Fig. by double integration method.



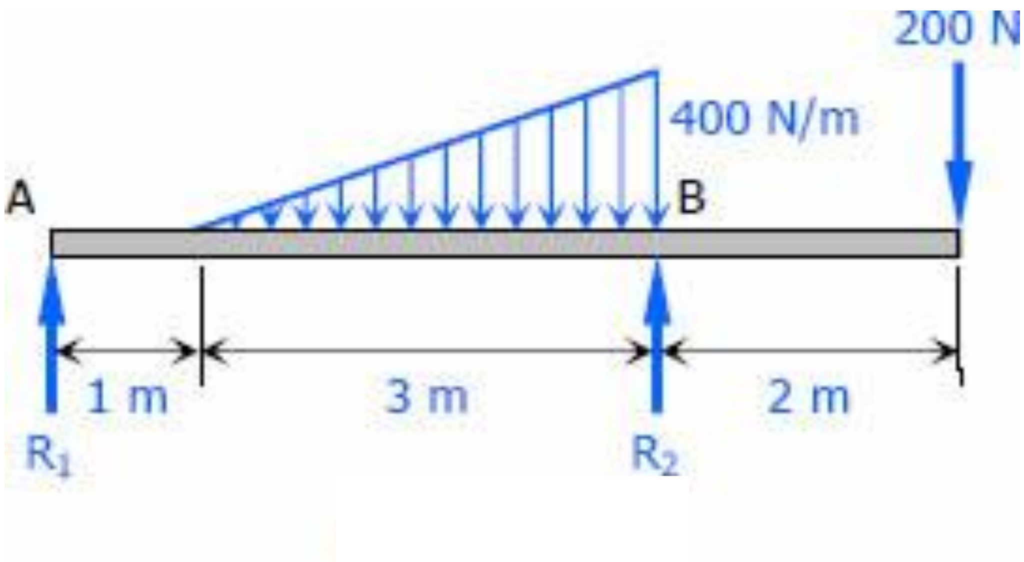
Problem# 620 (singer)

- Find the midspan deflection δ for the beam shown in Fig., carrying two triangularly distributed loads.

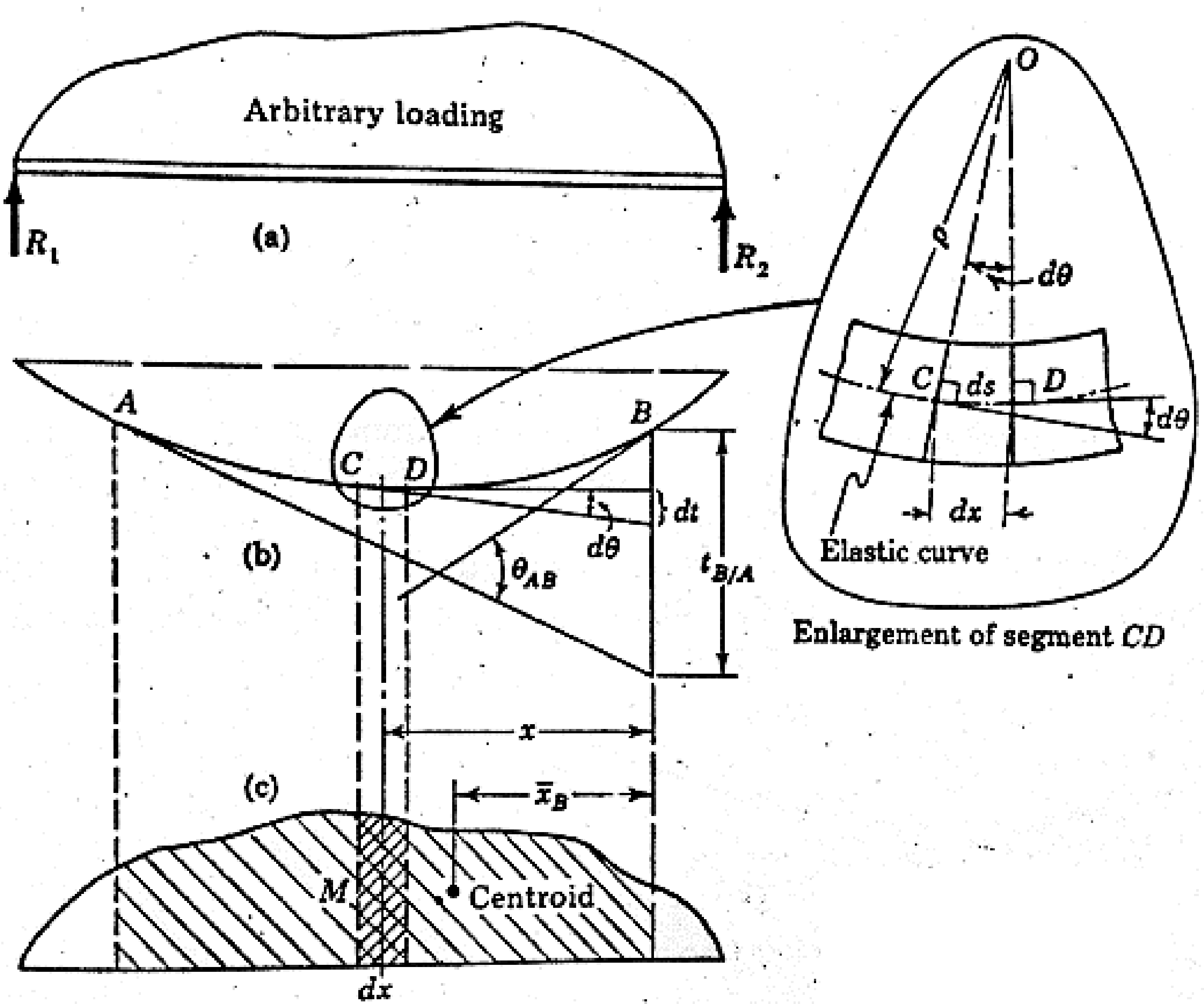


Problem# 630 (singer)

Determine the value of EIy midway between the supports for the beam loaded as shown in Fig. by double integration method.



Area-moment method



Area-moment method

We know, $\frac{1}{\rho} = \frac{M}{EI}$

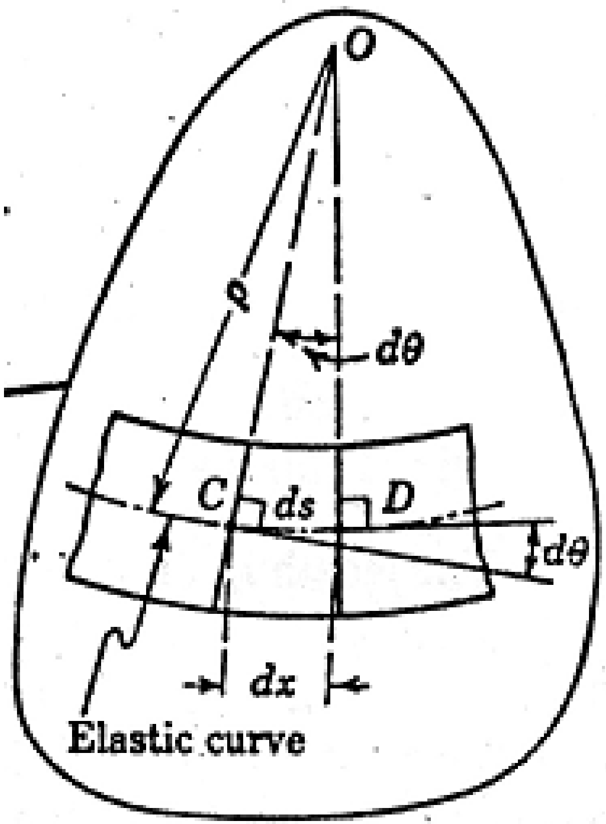
Since, $ds = \rho\, d\theta$, , we can write,

$$\frac{1}{\rho} = \frac{M}{EI} = \frac{d\theta}{ds}$$

Or, $d\theta = \frac{M}{EI} ds$

As the elastic curve is so flat that we can assume the length ‘ds’ to equal to its projection ‘dx’, we can write,

$$d\theta = \frac{M}{EI} dx$$



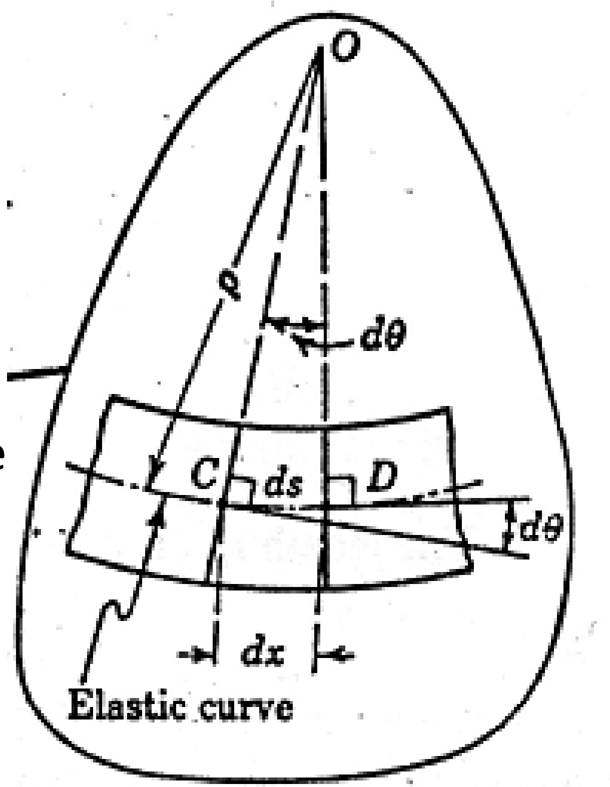
Enlargement of segment *CD*

Area-moment method

We get , $d\theta = \frac{M}{EI} dx$

As dθ is the angle between the tangents drawn to the elastic curve at C and D,

The change in slope between tangents drawn to the elastic curve at any two points A and B will equal to the sum of the small angles,



Enlargement of segment *CD*

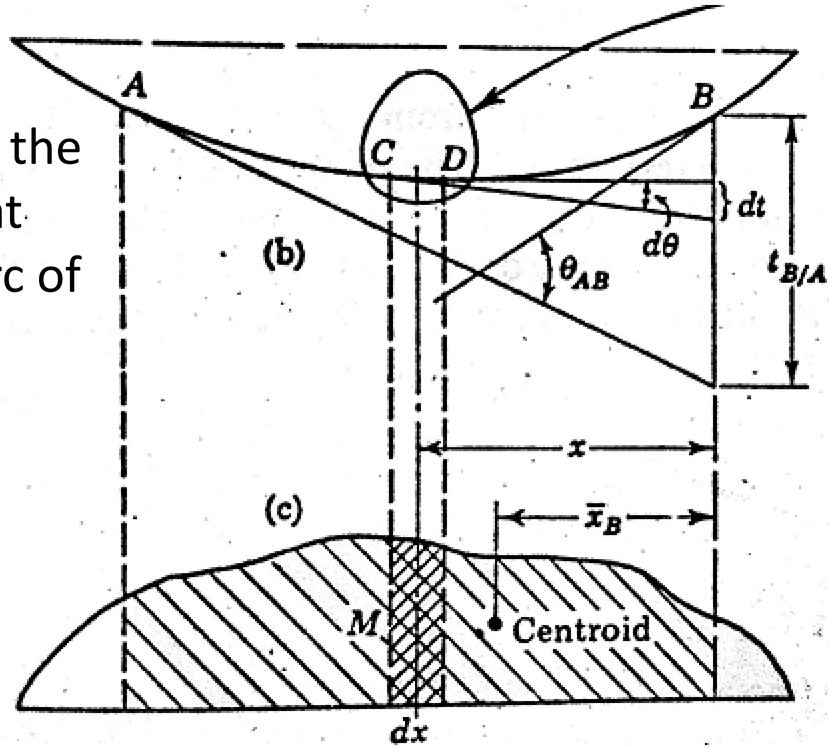
$$\theta_{AB} = \int_{\theta_A}^{\theta_B} d\theta = \frac{1}{EI} \int_{x_A}^{x_B} M\, dx$$

or,

$$\theta_{AB} = \frac{1}{EI}(\text{area})_{AB}$$

Area-moment method

The vertical distance from B on the elastic curve to the tangent drawn from point A is the sum of the intercepts ‘dt’ created by tangents to the curve at adjacent points. ‘dt’ may be considered as the arc of a circle of radius ‘x’ subtended by the angle dθ.



So, $$dt = x\,d\theta$$

Hence, $$t_{B/A} = \int dt = \int x\,d\theta$$

Replacing the value of dθ, we get

$$t_{B/A} = \frac{1}{EI}\int_{x_A}^{x_B} x(M\,dx)$$ or,

$$t_{B/A} = \frac{1}{EI}(\text{area})_{BA}\cdot \bar{x}_B$$

Area-moment method: Theorems

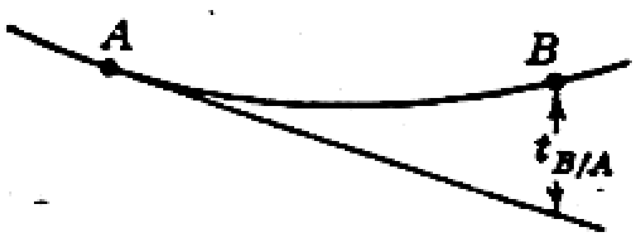
- **Theorem I :** The change in slope between the tangents drawn to the elastic curve at any two points A and B is equal to the product of 1/EI multiplied by the area of the moment diagram between these two points.

$$\theta_{AB} = \frac{1}{EI}(\text{area})_{AB}$$

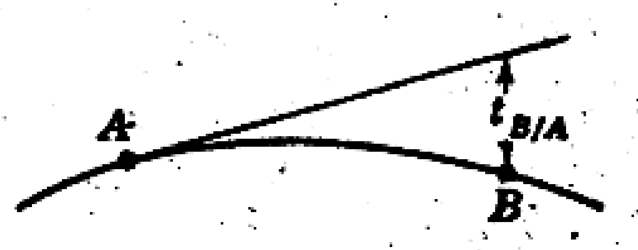
- **Theorem II :** The deviation of any point B relative to the tangent drawn to the elastic curve at any other point A, in a direction perpendicular to the original position of the beam, is equal to the product of 1/EI multiplied by the moment of an area about B of that part of the moment diagram between points A and B.

$$t_{B/A} = \frac{1}{EI}(\text{area})_{BA} \cdot \bar{x}_B$$

Area-moment method

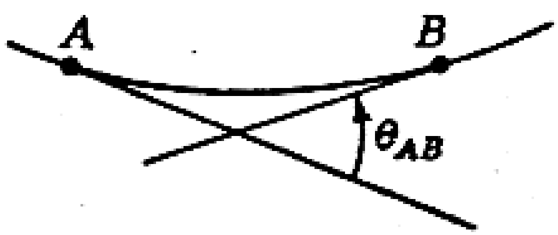


(a) Positive deviation; B located above reference tangent

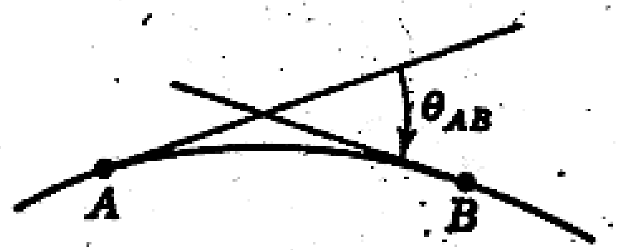


(b) Negative deviation; B located below reference tangent

Figure **Signs of deviations.**



(a) Positive change of slope; θ_{AB} is counterclockwise from left tangent

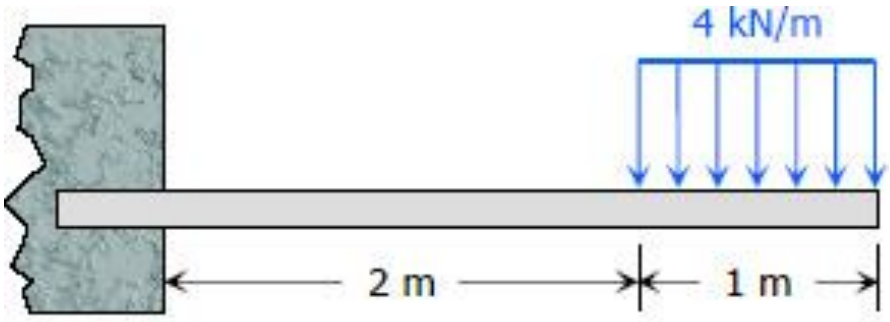


(b) Negative change of slope; θ_{AB} is clockwise from left tangent

Figure **Signs of change of slope.**

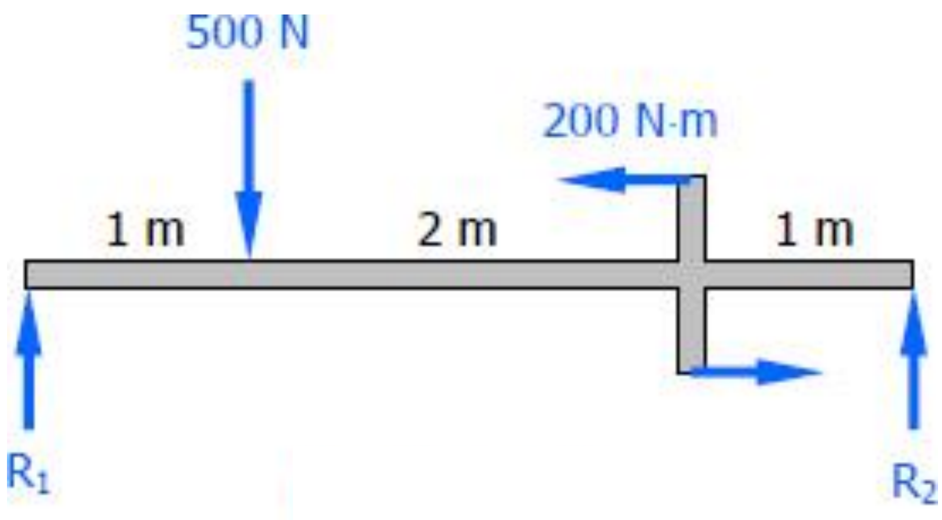
Problem# 642 (singer)

- Find the maximum deflection for the cantilever beam loaded as shown in Figure if the cross section is 50 mm wide by 150 mm high. Use E = 69 GPa.



Problem# 656 (singer)

- Find the value of EIδ at the point of application of the 200 N·m couple in Fig.



Problem# 667 (singer)

- Determine the value of EIδ at the right end of the overhanging beam shown in Fig. Is the deflection up or down?